Mathematics MDM4U Data Management
– Additional Exercise, Conditional Probability

– modified from Unionville High School MDM4U Handout

Level of Difficulty: Mandatory and Thinking and Inquiry

1. A bag contains white balls, a red ball and blue balls. There are four white balls labelled one to four. The red ball is marked with a five. The five blue balls are labelled six to ten. A ball is selected at random from the bag.
   a) What is the probability that the ball chosen is blue?
   b) If the ball chosen is known to be marked with an odd number, what is the probability that the ball is blue?

2. Two dice are rolled and the numbers on the up faces are observed. Find the probability of each of the following events.
   a) Exactly one die shows a 5, given the sum is seven.
   b) The sum of the numbers is seven, given that exactly one die shows a 5.
   c) The sum of the numbers is seven, given that at least one die shows a 5.

3. A nail manufacturer makes nails that are advertised to have a length of 15cm. A quality control test indicates that 71% of the products are 15cm, 18% of the product is less than 15cm, and 11% of the product is more than 15cm. Find, correct to 2 decimal places, the probability that a randomly selected nail that does not have a length of 15cm will have a length of less than 15cm.

4. You play a game of rolling two dice. You win the game if there is a red die while the sum is not seven (meaning '1' or '4'). As soon as a sum of seven appears, you lose. What is the probability of winning this game in one trial?

5. Two students are selected at random from a group consisting of five girls and three boys. Find the probability that both students are girls, given that at least one is a girl.

6. A single die is rolled. Find the probability of rolling each of the following.
   a) a two, given the number rolled is odd
   b) a four, given the number rolled is even
   c) an even number, give the number rolled is a 6

7. Two cards are drawn from a deck of 52 playing cards without any replacement. Find the probability of each of the following.
   a) both cards are clubs
   b) the second card is a club given the first one is a club
   c) the second card is black given the first one is a spade

8. A bag contains six green balls, and three black balls. A ball is selected without replacement, and then a second ball is selected. Find the probability of each of the following
   a) both balls are green
   b) both balls are black
   c) the second ball is green, given the first one is black
   d) one ball is black and the other one is green

9. Your class has won a homeroom contest and is rewarded with a pizza feast of twelve pizzas. Among the pizzas, two pizzas have bacon and anchovies, three pizzas have bacon and onions, three pizzas have green peppers and anchovies, and four pizzas have green peppers and onions. Two pizzas are randomly selected
   a) What is the probability that both pizzas have anchovies given that both have bacon?
   b) What is the probability that one pizza has anchovies and the other has onions given that both have bacon?

10. Three gamblers roll a die, in turn. The one who first rolls a 4 wins. What are their respective chances of winning on the first throw?

11. You have tickets to a rock concert. Three tickets are in the first row, two tickets are in the second row and four tickets are in the third row. You select two tickets at random to give to a friend. You notice that one of the tickets is not in the first row but you do not look at the other ticket. Find the probability of each of the following.
   a) both tickets are in the second row
   b) both tickets are in the third row

12. In your Finite Mathematics class, five students have programmable calculators, four have non-programmable calculators, and one has no calculator. Three students are selected at random from among these ten. All three have exactly the

same type of calculator. What is the probability that all three have programmable calculators?

13. The research branch of a pharmaceutical company did a study on the effect of a new painkilling drug. 500 people were tested. 250 were given the new drug, while 250were given a placebo. The results were as follows

	Felt better	Felt no better	Total
New Drug	185	65	250
Placebo	115	135	250
Total	300	200	500

a) What is the probability that a patient, selected at random, felt better after taking the new drug?
b) What is the probability that a patient, selected at random, received the placebo, given that he or she felt better?
c) What is the probability that a patient, selected at random, felt better, given that he or she received the placebo?

14. The gamblers toss a coin. The first one who tosses a tail wins. Show that the probability of the first gambler winning is $\frac{1}{2}+\frac{1}{16}+\frac{1}{128}...$

15. Four cards are drawn from a deck of 52 playing cards without any replacement. Find the probability of each of the following
a) all cards are hearts
b) all cards are hearts given two of them are hearts
c) all cards are spades given one of them was a diamond

16. A car insurance company examines the accident records of its policyholders for the past few years. It is found that 9.1% of the policyholders generally have had at least one accident, and 2.3% have had at least two accidents. What is the probability that a randomly selected policyholder, who has already had one accident during the present year, will have another accident this year?

A N S W E R S

1. See the following for details.
    a) Since there are five blue balls and a total of ten balls, the probability will be $5/10=1/2$, or simply 50%.
    b) Let event A be 'the ball is odd', and let event B be 'the ball is blue', we have $P(B|A)=P(B\cap A)/P(A)$, which is $(2/10)/(5/10)=2/5$, or simply 40%.

2. Let event A be 'exactly one 5', let event B be 'at least one 5', and let event C be 'the sum is 7'. We have …
    a) $P(A|C)=P(A\cap C)/P(C)$, which is $(2/36)/(6/36)=1/3$.
    b) $P(C|A)=P(C\cap A)/P(A)$, which is $(2/36)/(10/36)=1/5$.
    c) $P(C|B)=P(C\cap B)/P(B)$, which is $(2/36)/(11/36)=2/11$.

3. Let event A be 'not does have 15cm', and let event B be 'length less than 15cm'. We have $P(B|A)=P(B\cap A)/P(A)$, which is $0.18/0.29=62\%$.

4. Let even A be 'rolling a 1 or a 6', and let event B be 'not rolling a sum of seven'. We have $P(A|B)=P(A\cap B)/P(B)$. For event B, by applying the indirect method, we can count there are $36-6=30$ ways. So, $P(B)=30/36$. In order for event A to happen, we have $12+4\times 2=20$ ways. Since there are four ways in which A and B can happen at the same time, namely 16, 61, 34, and 43, we know that $P(A\cap B)=16/36$. Therefore, we have our final answer $(16/36)/(30/36)=8/15$.

5. Let event A be 'both are girls', and let event B be 'at least one girl'. We have $P(A|B)=P(A\cap B)/P(B)$. $P(A\cap B)=P(A)=C(5,2)/C(8,2)=10/28$, and we also have $P(B)=1-C(3,2)/C(8,2)=25/28$. Therefore, we have the final answer being $(10/28)/(25/28)=2/5$, or 40%.

6. For the following, let event A be 'rolling a 2', let event B be 'rolling a 4', let event C be 'rolling a 6', let event D be 'rolling an odd number', and let event E be 'rolling an even number'. We have …
    a) $P(A|D)=P(A\cap D)/P(D)=0$, because $P(A\cap D)=0$.
    b) $P(B|E)=P(B\cap E)/P(E)$, which is $(1/6)/(3/6)=1/3$.
    c) $P(E|C)=P(E\cap C)/P(C)$, which is $(3/6)/(1/6)=3$. According to the formula, we obtain a 300% probability. However, probability cannot

exceed 100%. Thus, the answer should be 100%. Logically, one should also be able to answer that the probability is 100%, because given the die rolled is already six, then there will be no doubt that the rolled number is even. Thus, we may logically conclude the probability being 100%.

7. Let event A be 'both cards are clubs', event B be 'the first card is a club', let event C be 'the second card is a black', and let event D be 'the first card is a spade'. Thus, we have
   a) $P(A) = P(13,2)/P(52,2) = 1/17$
   b) $P(A|B) = P(A \cap B)/P(B)$, since $P(A \cap B) = P(A) = 1/17$, we have $(1/17)/(13/52) = 4/17$, or in percentage approximately 23.5%.
   c) $P(C|D) = P(C \cap D)/P(D)$. By analyzing with a tree diagram, we find that $P(C \cap D) = (13 \times 25)/(52 \times 51) = 25/204 = 12.3\%$. Since we know $P(D) = 13/52$, then $(25/204)/(13/52) = 25/51 \approx 49\%$.

8. See the following for details.
   a) If both were green, then $P(6,2)/P(9,2) = 5/12 = 41.67\%$.
   b) If both were black, then $P(3,2)/P(9,2) = 1/12 = 8.33\%$.
   c) Let event A be 'the second ball is green', and let event B be 'the first one is black'. $P(A|B) = P(A \cap B)/P(B)$, where we first calculate $P(A \cap B) = P(3,1)P(6,1)/P(9,2) = 1/4 = 25.0\%$. Thus, our final answer will be $(1/4)/(3/9) = 3/4 = 75.0\%$.
   d) This question will be solve by two distinct methods:
      i. Having one green one black is the same as saying either first black second green, or first green second black. Thus, it will be $P(3,1)P(6,1)/P(9,2) = 1/4$ and $P(6,1)P(3,1)/P(9,2) = 1/4$, which will be $(1/4)+(1/4) = 1/2 = 50\%$.
      ii. Having a color each is the same as saying no two greens and no two blacks. If we apply the indirect method, we see the probability of two green balls will be $P(6,2)/P(9,2) = 5/12$, while the probability of selecting two black balls will be $P(3,2)/P(9,2) = 1/12$. Therefore, the final answer will be 100% subtract the sum of both events $1 - 5/12 - 1/12 = 50\%$.

9. Let event A be 'both pizzas have anchovies', let event B be 'both pizzas have bacon', let event C be 'one pizza has anchovies and the other has onions'. Thus, we have …
   a) $P(A|B) = P(A \cap B)/P(B)$. There are two pizzas with both anchovies and

bacon. Thus, the probability of $P(A \cap B)$ will be by combinations, $C(2,2)/C(12,2) = 1/66 = 1.52\%$. There are five pizzas with bacon. Thus, $P(B) = C(5,2)/C(12,2) = 10/66 = 15.15\%$. Therefore, the final answer will be $(1/66)/(10/66) = 1/10 = 10\%$.

b) $P(C \mid B) = P(C \cap B)/P(B)$. $P(C \cap B)$ is not so straightforward; we can interpret this as 'the probability of having one pizza with anchovies and bacon, while the other has onions and bacon'. Thus, with organized counting we see $P(C \cap B) = C(2,1)C(3,1)/C(12,2)$, which gives $1/11 = 9.1\%$. Since $P(B) = 10/66$, we have the final answer $(1/11)/(10/66) = 3/5 = 60\%$.

Note: textbook answer was $3/10 = 30\%$

10. The first gambler, obviously, has $1/6 \approx 16.67\%$ chances of winning. The second gambler only has a chance to win if the first gambler did not roll a four. Thus, his chances of winning will be $(5/6)(1/6) = 5/36 = 13.89\%$. The third gambler's chance will be $(5/6)^2(1/6) = 25/216 \approx 11.57\%$.

11. Let event A be 'one ticket is not in the first row', let event B be 'both tickets are in the second row', and let event C be 'both tickets are in the third row'. Thus, we have …

a) First, we find $P(A \cap B) = C(2,2)/C(9,2) = 1/36 \approx 2.78\%$. For one of the tickets not in the first row, we may find $P(A) \approx 91.67\%$. This is obtained by $P(A) = [C(3,1)C(6,1) + C(6,2)]/C(9,2) = 11/12$. One may attempt to use an indirect method, which gives $P(A) = 1 - C(6,2)/C(9,2) \approx 91.67\%$. Therefore, $P(B \mid A) = (1/36)/(11/12) = 1/33 \approx 3.33\%$.

b) Similarly, we may find that $P(A \cap C) = C(4,2)/C(9,2) = 1/6 \approx 16.67\%$. The probability for $P(A) \approx 91.67\%$ is still the same. Thus, we have $P(C \mid A) = (1/6)/(11/12) = 2/11 \approx 18.18\%$.

12. Let event A be 'all three students have exactly the same type of calculators', and event B be 'all three students have programmable calculators'. We find that $P(B \mid A) = P(A \cap B)/P(A)$. For both events to happen at the same time, we find $P(A \cap B) = C(5,3)/C(10,3) = 1/12 \approx 8.33\%$. To have the same type of calculators, there are only two possible ways – all programmable, or all non-programmable. Thus, $P(A) = [C(5,3) + C(4,3)]/C(10,3) = 7/60 \approx 11.67\%$. Therefore, $P(B \mid A) = (1/12)/(7/60) = 5/7 \approx 71.43\%$.

13. See the following for details.
    a) from the chart, we have $185/250 = 37/50 = 74\%$
    b) from the chart, we have $115/300 = 23/60 \approx 38.33\%$
    c) from the chart, we have $115/250 = 23/50 = 46\%$

14. Ask Mr. Gao for the full explanation

15. Let event A be ‘all four cards are hearts’, event B be ‘two of the cards are hearts’, event C be ‘all cards are spades’, and event D be ‘one of the cards is a diamond’. Under these definitions, we have …
    a) $P(A) = C(13,4)/C(52,4) = 11/4165 \approx 0.264\%$
    b) $P(A \cap B) = C(13,4)/C(52,4) = 11/4165 \approx 0.264\%$. On the other hand, we may find that two of the cards are hearts means that there might be B_1 'exactly two hearts’, or B_2 'exactly three hearts’, or B_3 'exactly four hearts. As a result, we see that $P(B) = P(B_1) + P(B_2) + P(B_3)$. So, separately, we calculate

$$P(B_1) = C(13,2)C(39,2)/C(52,4) = 4446/20825 \approx 21.35\%$$
$$P(B_2) = C(13,3)C(39,1)/C(52,4) = 858/20825 \approx 4.12\%$$
$$P(B_3) = C(13,4)C(39,0)/C(52,4) = 55/20825 \approx 0.264\%$$

    Therefore, we find $P(B) = 1079/4165 \approx 25.91\%$. At last, we find $P(A \mid B) = (11/4165)/(1079/4165) = 11/1079 \approx 25.91\%$.
    c) C and D are mutually exclusive events; they cannot happen at the same time. Therefore, $P(C \mid D) = 0\%$.

16. Let event A be ‘have one accident’, and let event B be ‘have two accidents’. We have $P(B \mid A) = 2.3\%/9.1\% \approx 25.27\%$.